QED $\longrightarrow$ Operador de Scattering a orden 2

Necesitamos:

1) Expansiones de Fourier de los campos.
2) Relaciones de (anti-) conmutación.
3) Lagrangiano de interacción.

- El campo de Dirac

on-shell !

$$\Psi_\alpha(x) = \frac{1}{(2\pi)^{3/2}} \sum_{\sigma=1}^{2} \int \frac{d^3\vec{p}}{2E_p} \left(a_\sigma(p)\, u_\alpha^{(\sigma)}(p)\, e^{-ip\cdot x} + b_\sigma^\dagger(p)\, v_\alpha^{(\sigma)}(p)\, e^{ip\cdot x} \right)$$

on-shell !

$$\overline{\Psi}_\alpha(x) = \frac{1}{(2\pi)^{3/2}} \sum_{\sigma=1}^{2} \int \frac{d^3\vec{p}}{2E_p} \left(a_\sigma^\dagger(p)\, \overline{u_\alpha^{(\sigma)}}(p)\, e^{ip\cdot x} + b_\sigma(p)\, \overline{v_\alpha^{(\sigma)}}(p)\, e^{-ip\cdot x} \right)$$

$$\overline{\Psi} = \Psi^\dagger \gamma^0 , \qquad (\not{p} - m)\, u^{(\sigma)}(p) = 0 , \qquad (\not{p} + m)\, v^{(\sigma)}(p) = 0 .$$

Relaciones de anticonmutación:

$$\{ a_\sigma(p), a_{\sigma'}^\dagger(q) \} = 2E_p\, \delta_{\sigma\sigma'}\, \delta(\vec{p} - \vec{q}) , \qquad \{ b, b^\dagger \} = 2E\delta \ldots , \quad \text{etc.}$$

Propagador:

$$S_F(x-y) = \langle 0 | \tilde{T}(\Psi(x)\overline{\Psi}(y)) | 0 \rangle = \frac{i}{(2\pi)^4} \int d^4k \, \frac{(\not{k} + m)\, e^{-ik\cdot(x-y)}}{k^2 - m^2 + i\varepsilon}$$

- El campo electromagnético:

$$A_\mu(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3\vec{k}}{2\omega_k} \sum_{\lambda=0}^{3} \left(\varepsilon_\mu^{(\lambda)}(k)\, c_\lambda(k)\, e^{-ik\cdot x} + \varepsilon_\mu^{(\lambda)*}(k)\, c_\lambda^\dagger(k)\, e^{ik\cdot x} \right)$$

CCR: $\left[c^{(\lambda)}(k), c^{(\lambda')\dagger}(k') \right] = -2\omega_k\, g^{\lambda\lambda'}\, \delta(k - k')$

Condición de Lorenz $\longrightarrow$ "$\langle \Psi | \partial_\mu A^\mu(x) | \Psi \rangle = 0$" , $|\Psi\rangle$: "estado físico"

$$\Rightarrow \left(c^{(0)}(k) - c^{(3)}(k) \right) |\Psi\rangle = 0 .$$

$$\mathcal{L} = \mathcal{L}_{EM} - \frac{\Lambda}{2} \left(\partial_\mu A^\mu \right)^2$$

"gauge fixing term"

$$\hookrightarrow \langle 0|T(A^\mu(x)A^\nu(y))|0\rangle = -\frac{i}{(2\pi)^4}\int d^4k \frac{e^{-ik\cdot(x-y)}}{k^2+i\varepsilon}\left(g^{\mu\nu}+\frac{(1-\Lambda)}{\Lambda}\frac{k^\mu k^\nu}{k^2}\right)$$

$\Lambda = 1 \longrightarrow$ "gauge de Feynman"

<u>Propagador:</u>

$$D_F^{\mu\nu}(x-y) = \langle 0|T(A^\mu(x)A^\nu(y))|0\rangle$$

- <u>Lagrangiano de interacción:</u>

$$\mathcal{L}_I = -e :\overline{\Psi}(x)\gamma^\mu\Psi(x)A_\mu(x):, \text{ donde } e = \text{carga del electrón } (e<0).$$

$$\searrow \mathcal{H}_I(x) = e :\overline{\Psi}(x)\gamma^\mu\Psi(x)A_\mu(x): \longrightarrow H_I = \int d^3\vec{x}\, \mathcal{H}_I(x).$$

Serie de Dyson:

$$\mathcal{S} = \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int dx_1\cdots dx_n\, T(\mathcal{H}_I(x_1)\cdots\mathcal{H}_I(x_n)) \equiv \sum_{n=0}^{\infty}\mathcal{S}^{(n)}$$

Orden 2 $\longrightarrow \mathcal{S}^{(2)}$

$$\mathcal{S}^{(2)} = \frac{(-ie)^2}{2!}\int d^4x\int d^4y\, T\left(:\overline{\Psi}(x)\gamma^\mu\Psi(x)A_\mu(x)::\overline{\Psi}(y)\gamma^\nu\Psi(y)A_\nu(y):\right)$$

$$= \frac{(-ie)^2}{2!}\int d^4x\int d^4y\, T\left(:\overline{\Psi}(x)\gamma^\mu\Psi(x)::\overline{\Psi}(y)\gamma^\nu\Psi(y):\right)T(A_\mu(x)A_\nu(y))$$

$\longrightarrow$ el operador T se puede separar de esta forma debido a que los Ψ's y los A's conmutan entre sí.

$\longrightarrow$ Calculamos los diferentes términos por aparte.

$$\boxed{T(A_\mu(x)A_\nu(y))}$$

$$T(A_\mu(x)A_\nu(y)) = :A_\mu(x)A_\nu(y): + \langle 0|T(A_\mu(x)A_\nu(y))|0\rangle$$

$$= :A_\mu(x)A_\nu(y): + D_{\mu\nu}(x-y)$$

$$\boxed{T\left(:\bar{\psi}(x)\gamma^\mu\psi(x)::\bar{\psi}(y)\gamma^\nu\psi(y):\right)}$$

$$T\left(:\bar{\psi}(x)\gamma^\mu\psi(x)::\bar{\psi}(y)\gamma^\nu\psi(y):\right) =$$

$$= (\gamma^\mu)_{\alpha\beta}(\gamma^\nu)_{\sigma\tau}\,\tilde{T}\left(:\bar{\psi}_\alpha(x)\psi_\beta(x)::\bar{\psi}_\sigma(y)\psi_\tau(y):\right)$$

aquí podemos usar el operador de ordenamiento temporal "fermiónico", porque tenemos números pares de operadores fermiónicos.

$$= (\gamma^\mu)_{\alpha\beta}(\gamma^\nu)_{\sigma\tau}\Big(:\bar{\psi}_\alpha(x)\psi_\beta(x)\bar{\psi}_\sigma(y)\psi_\tau(y): + :\wick{\c1{\bar{\psi}_\alpha(x)}\psi_\beta(x)\bar{\psi}_\sigma(y)\c1{\psi_\tau(y)}}:$$

$$+ :\bar{\psi}_\alpha(x)\wick{\c1{\psi_\beta(x)}\c1{\bar{\psi}_\sigma(y)}}\psi_\tau(y): + :\wick{\c1{\bar{\psi}_\alpha(x)}\c2{\psi_\beta(x)}\c2{\bar{\psi}_\sigma(y)}\c1{\psi_\tau(y)}}:\Big)$$

$$\stackrel{(*)}{=} (\gamma^\mu)_{\alpha\beta}(\gamma^\nu)_{\sigma\tau}\Big(:\bar{\psi}_\alpha(x)\psi_\beta(x)\bar{\psi}_\sigma(y)\psi_\tau(y) - S_{\tau\alpha}(y-x):\psi_\beta(x)\bar{\psi}_\sigma(y): + S_{\beta\sigma}(x-y):\bar{\psi}_\alpha(x)\psi_\tau(y):$$

$$- S_{\tau\alpha}(y-x)\,S_{\beta\sigma}(x-y)\Big)$$

$$= :\bar{\psi}(x)\gamma^\mu\psi(x)\bar{\psi}(y)\gamma^\nu\psi(y): + :\bar{\psi}(y)\gamma^\nu S(y-x)\gamma^\mu\psi(x):$$

$$+ :\bar{\psi}(x)\gamma^\mu S(x-y)\gamma^\nu\psi(y): - \operatorname{tr}\left(S(y-x)\gamma^\mu S(x-y)\gamma^\nu\right).$$

(*) → Aquí hemos hecho uso de la identidad

$$\wick{\c1{\bar{\psi}_\alpha(x)}\c1{\psi_\beta(y)}} = -\wick{\c1{\psi_\beta(y)}\c1{\bar{\psi}_\alpha(x)}},$$

que a su vez es consecuencia de:

$$\tilde{T}(AB) = :AB: + \wick{\c1A\c1B}$$

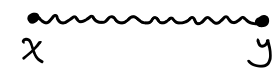
$$\tilde{T}(BA) = :BA: + \wick{\c1B\c1A} \quad \rightsquigarrow \quad \tilde{T}(AB) + \tilde{T}(BA) = AB\,\theta(t_A - t_B) - BA\,\theta(t_B - t_A)$$

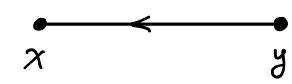
$$:AB: = -:BA: \qquad\qquad + BA\,\theta(t_B - t_A) - AB\,\theta(t_A - t_B)$$

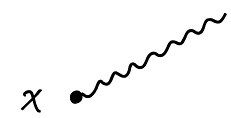
$$= 0$$

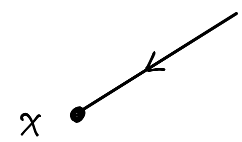
$$\Rightarrow \quad 0 = \wick{\c1A\c1B} + \wick{\c1B\c1A}$$

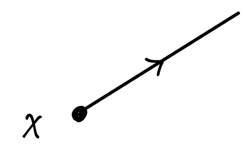
Reglas (preliminar):

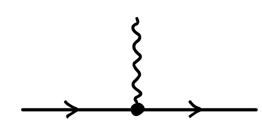
$A_\mu(x)\,A_\nu(y)$ $\longleftrightarrow$ 

$\Psi_\mu(x)\;\overline{\Psi}_\nu(y)$ $\longleftrightarrow$ 

$A_\mu(x)$ (libre) $\longleftrightarrow$ 

$\Psi(x)$ (libre) $\longleftrightarrow$ 

$\overline{\Psi}(x)$ (libre) $\longleftrightarrow$ 

$e\gamma^\mu$ (vértice) $\longleftrightarrow$ 

Juntando lo que tenemos:

$$\text{“}\, T(:\overline{\Psi}\gamma\Psi::\overline{\Psi}\gamma\Psi:)\,T(AA)\,\text{”} =$$

$$= \Big(:\overline{\Psi}(x)\gamma^\mu\Psi(x)\overline{\Psi}(y)\gamma^\nu\Psi(y): + :\overline{\Psi}(y)\gamma^\nu S(y-x)\gamma^\mu\Psi(x):$$

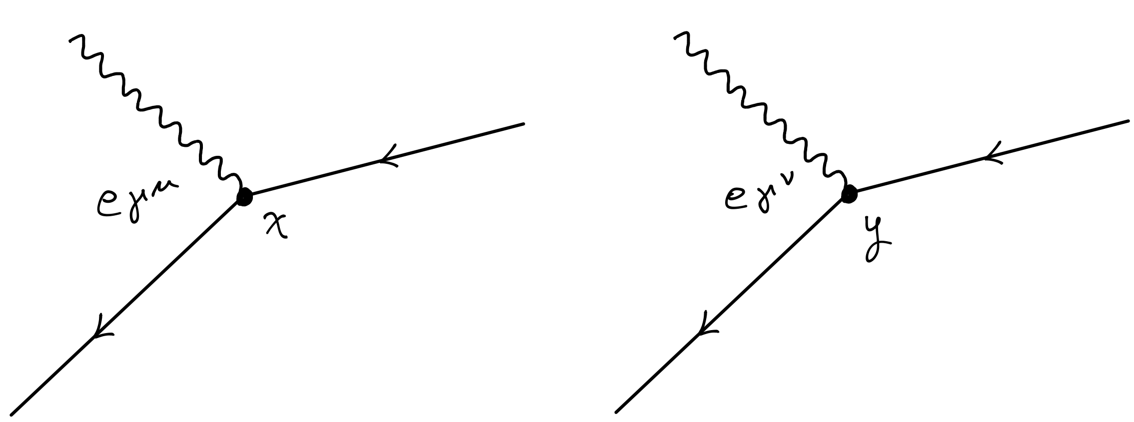
$$+ :\overline{\Psi}(x)\gamma^\mu S(x-y)\gamma^\nu\Psi(y): - \mathrm{tr}\big(S(y-x)\gamma^\mu S(x-y)\gamma^\nu\big)\Big)\times$$

$$\times\Big(:A_\mu(x)A_\nu(y): + D_{\mu\nu}(x-y)\Big)$$

$\longrightarrow$ 8 términos en total.

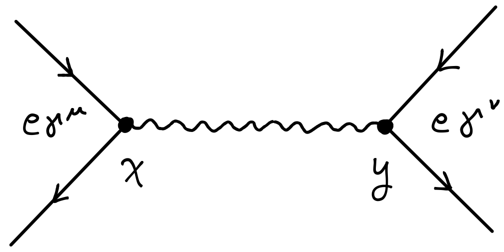
Representación gráfica $\longrightarrow$

(1) $:\overline{\Psi}(x)\gamma^\mu\Psi(x)\overline{\Psi}(y)\gamma^\nu\Psi(y)::A_\mu(x)A_\nu(y):$



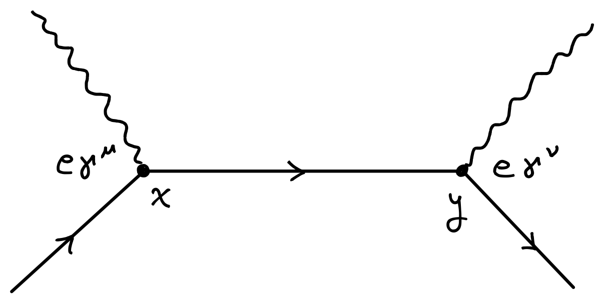
(disconexos)

② $:\overline{\Psi}(x)\,\gamma^\mu\,\Psi(x)\,\overline{\Psi}(y)\,\gamma^\nu\,\Psi(y):\,D_{\mu\nu}(x-y)$

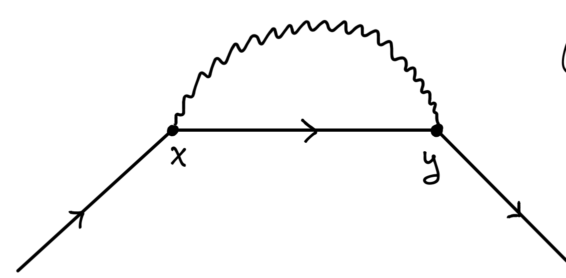


(ejemplo: scattering $e^- - e^-$)

③ $:\overline{\Psi}(y)\,\gamma^\nu\,S(y-x)\,\gamma^\mu\,\Psi(x):\ :A_\mu(x)\,A_\nu(y):$ (ejemplo: $e^- + \gamma \rightarrow e^- + \gamma$)

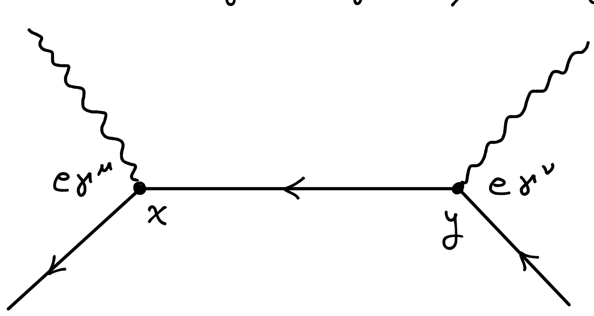


④ $:\overline{\Psi}(y)\,\gamma^\nu\,S(y-x)\,\gamma^\mu\,\Psi(x):\,D_{\mu\nu}(x-y)$

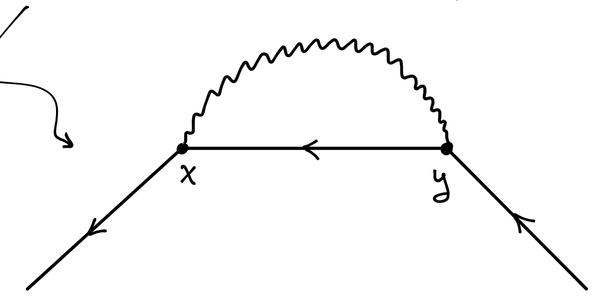


(autoenergía)

⑤ $:\overline{\Psi}(x)\,\gamma^\mu\,S(x-y)\,\gamma^\nu\,\Psi(y):\ :A_\mu(x)\,A_\nu(y):$

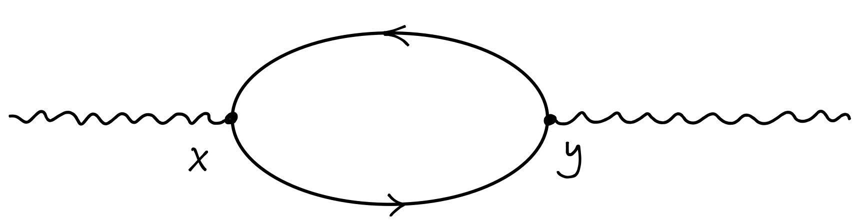


⑥ $:\overline{\Psi}(x)\,\gamma^\mu\,S(x-y)\,\gamma^\nu\,\Psi(y):\,D_{\mu\nu}(x-y)$



⑦ $-\mathrm{tr}\left(S(y-x)\,\gamma^\mu\,S(x-y)\,\gamma^\nu\right):A_\mu(x)\,A_\nu(y):$

(autoenergía - fotón)



⑧ $-\mathrm{tr}\left(S(y-x)\,\gamma^\mu\,S(x-y)\,\gamma^\nu\right)D_{\mu\nu}(x-y)$

(diagrama de vacío)